# SINGULARLY PERTURBED QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS ON THE PLANE

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▪ *ABSTRACT: In this paper singularly perturbed quadratic polynomial differential systems defined in $\mathbb{R}^2$ are discussed. The main results describe the fast and the slow dynamics of a list of quadratic systems obtained by singular perturbation of normal forms. In particular we show how limit cycles can appear perturbing singular quadratic systems.*

▪ *KEYWORDS: Limit cycles; vector fields; singular perturbation; quadratic systems.*

## 1 Introduction and statement of the main results

The present work fits within the study of singularly perturbed quadratic systems:

$$x' = \frac{dx}{d\tau} = p_\varepsilon(x,y) = p(x,y,\varepsilon), \quad y' = \frac{dy}{d\tau} = \varepsilon q_\varepsilon(x,y) = \varepsilon q(x,y,\varepsilon), \tag{1}$$

with $p, q$ real polynomials, $p_\varepsilon, q_\varepsilon$ quadratic in $x = x(\tau)$ and $y = y(\tau) \in \mathbb{R}, \varepsilon \geqslant 0$ and $(p_\varepsilon, q_\varepsilon) = 1$ for $\varepsilon > 0$ small.

The techniques of the *Geometric Singular Perturbation theory* (GSP–theory) can be used to obtain information on the dynamics of system (1) for small values of $\varepsilon > 0$, specially in the search of limit cycles.

As usual in GSP–theory, we consider the time rescaling $t = \varepsilon\tau$ to get

$$\varepsilon\dot{x} = p_\varepsilon(x,y), \quad \dot{y} = q_\varepsilon(x,y), \tag{2}$$

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