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Global Phase Portraits of Kukles Differential Systems with Homogeneous Polynomial Nonlinearities of Degree 6 Having a Center and Their Small Limit Cycles

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We provide the nine topological global phase portraits in the Poincaré disk of the family of the centers of Kukles polynomial differential systems of the form $\dot{x} = -y$, $\dot{y} = x + ax^5y + bx^3y^3 + cxy^5$, where $x, y \in \mathbb{R}$ and a, b, c are real parameters satisfying $a^2 + b^2 + c^2 \neq 0$. Using averaging theory up to sixth order we determine the number of limit cycles which bifurcate from the origin when we perturb this system first inside the class of all homogeneous polynomial differential systems of degree 6, and second inside the class of all polynomial differential systems of degree 6.

Keywords: Center; Kukles; polynomial vector fields; Poincaré disk; averaging.

1. Introduction and Main Results

Consider the planar differential systems

$$\dot{x} = -y, \quad \dot{y} = x + Q_n(x, y), \tag{1}$$

where Q_n is a real homogeneous polynomial of degree n. Here the dot denotes the derivative with respect to the independent variable t. Giné [2002] called the systems of this type *Kukles homogeneous systems*.

Volokitin and Ivanov [1999] conjectured that, for $n \geq 2$, system (1) has a center at the origin if, and only if, this system is symmetric with respect to one of the coordinate axes. For $n = 2$ and $n = 3$, the authors knew that the answer was positive. Giné [2002] proved that this conjecture is true for $n = 4$ and $n = 5$. In [Giné *et al.*, 2015a, 2015b], the authors proved that the answer to this question is positive for an arbitrary n odd and an arbitrary n even, respectively.

Here we use the work [Giné *et al.*, 2015b] to present the topological classification in the Poincaré disk of the phase portraits of the planar system

$$\dot{x} = -y, \quad \dot{y} = x + ax^5y + bx^3y^3 + cxy^5, \tag{2}$$

where a, b and c are real coefficients with $a^2 + b^2 + c^2 \neq 0$ and $(x, y) \in \mathbb{R}^2$. Since system (2) is invariant under the change of coordinates $(t, x, y) \mapsto (-t, -x, y)$, its phase portrait is symmetric with respect to the y-axis, then it follows that the origin is a center of system (2).