Regularization of Discontinuous Vector Fields on $\mathbb{R}^3$ via Singular Perturbation

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Singular perturbations problems in dimension three which are approximations of discontinuous vector fields are studied in this paper. The main result states that the regularization process developed by Sotomayor and Teixeira produces a singular problem for which the discontinuous set is a center manifold. Moreover, the definition of sliding vector field coincides with the reduced problem of the corresponding singular problem for a class of vector fields.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Let $F: (\mathbb{R}^3, 0) \longrightarrow (\mathbb{R}, 0)$ be a C^∞ function having $0 \in \mathbb{R}$ as a regular value (i.e., $\nabla F(p) \neq 0$, for any $p \in F^{-1}(0)$). We denote $F^{-1}(0)$ by Σ. We write $\Omega^r = \Omega^r(\mathbb{R}^3, F)$ the space of vector fields $X: \mathbb{R}^3 \setminus \Sigma \longrightarrow \mathbb{R}^3$ such that

$$X(q) = \begin{cases} X_1(q), & \text{if } F(q) \geqslant 0, \\ X_2(q), & \text{if } F(q) \leqslant 0, \end{cases} \tag{1}$$

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