Sliding Vector Fields via Slow–Fast Systems

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Dedicated to Freddy Dumortier for his 60–th birthday.

Abstract

This paper concerns differential equation systems on $\mathbb{R}^n$ with discontinuous right–hand sides. We deal with non-smooth vector fields in $\mathbb{R}^n$ having a codimension-one submanifold M as its discontinuity set. After a regularization of a such system and a global blow–up we are able to bring out some results that bridge the space between discontinuous systems and singularly perturbed smooth systems.

1 Introduction and statement of the main results

In this paper a discussion is focused to study the phase portraits of certain non–smooth vector fields defined in $\mathbb{R}^n$ having a codimension–one submanifold M as its discontinuity set. We present some results in the framework developed by Sotomayor and Teixeira in [9] and establish a bridge between those systems and the fundamental role played by the Geometric Singular Perturbation Theory (GSPT). This transition was introduced in [1] and [7], in dimensions 2 and 3, respectively. Needless to say that in this area very good surveys are available (see [2, 3, 5, 6] for instance).

Let $U \subseteq \mathbb{R}^n$ be an open set. We suppose that $M = F^{-1}(0)$ where $F : U \to \mathbb{R}$ is a smooth function and $0 \in \mathbb{R}$ is a regular value of F. Clearly M is the separating boundary of the regions $M_+ = \{q \in U | F(q) \geq 0\}$ and $M_- = \{q \in U | F(q) \leq 0\}$. We denote by $\mathcal{C}^r(U, \mathbb{R}^n)$ the set of all vector fields of class $\mathcal{C}^r$ defined on U, with $r \geqslant 1$, endowed with the C^r–topology.

Denote by $\Omega^r(U)$ the set of all vector fields X on U such that

$$X(q) = \begin{cases} X_1(q) & \text{if} \quad q \in M_+, \\ X_2(q) & \text{if} \quad q \in M_-, \end{cases} \tag{1}$$

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