

# Synchronization and Non-Smooth Dynamical Systems

Jaume Llibre · Paulo R. da Silva · Marco A. Teixeira

Received: 12 June 2011 / Revised: 30 October 2011 / Published online: 4 February 2012  
© Springer Science+Business Media, LLC 2012

**Abstract** In this article we establish an interaction between non-smooth systems, geometric singular perturbation theory and synchronization phenomena. We find conditions for a non-smooth vector fields be locally synchronized. Moreover its regularization provide a singular perturbation problem with attracting critical manifold. We also state a result about the synchronization which occurs in the regularization of the fold-fold case. We restrict ourselves to the 3-dimensional systems ( $\ell = 3$ ) and consider the case known as a *T-singularity*.

**Keywords** Regularization · Vector field · Constrained system · Singular perturbation · Non-smooth vector field · Sliding vector field · Synchronization

**Mathematics Subject Classification (1991)** Primary 34C20 · 34C26 · 34D15 · 34H05

## 1 Introduction

The main goal of this paper is to establish an interaction between three important themes of the qualitative theory of non-smooth dynamical systems:

- synchronization phenomena,
- sliding vector fields (also known as Filippov systems) and
- singular perturbation.

---

J. Llibre  
Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona,  
Catalonia, Spain  
e-mail: jllibre@mat.uab.cat

P. R. da Silva (✉)  
Departamento de Matemática, IBILCE–UNESP, Rua C. Colombo, 2265, S. J. Rio Preto,  
São Paulo 15054–000, Brazil  
e-mail: prs@ibilce.unesp.br

M. A. Teixeira  
IMECC–UNICAMP, Campinas, São Paulo 13081–970, Brazil  
e-mail: teixeira@ime.unicamp.br