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Sliding vector fields for non-smooth dynamical systems having intersecting switching manifolds

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Abstract

We consider a differential equation $\dot{p} = X(p)$, $p \in \mathbb{R}^3$, with discontinuous right-hand side and discontinuities occurring on a set Σ. We discuss the dynamics of the sliding mode which occurs when, for any initial condition near $p \in \Sigma$, the corresponding solution trajectories are attracted to Σ. Firstly we suppose that $\Sigma = H^{-1}(0)$, where H is a smooth function and $0 \in \mathbb{R}$ is a regular value. In this case Σ is locally diffeomorphic to the set $\mathcal{F} = \{(x, y, z) \in \mathbb{R}^3; z = 0\}$. Secondly we suppose that Σ is the inverse image of a non-regular value. We focus our attention to the equations defined around singularities as described in Gutierrez and Sotomayor (1982 *Proc. Lond. Math. Soc* **45** 97–112). More precisely, we restrict the degeneracy of the singularity so as to admit only those which appear when the regularity conditions in the definition of smooth surfaces of $\mathbb{R}^3$ in terms of implicit functions and immersions are broken in a stable manner. In this case Σ is locally diffeomorphic to one of the following algebraic varieties: $\mathcal{D} = \{(x, y, z) \in \mathbb{R}^3; xy = 0\}$ (double crossing); $\mathcal{T} = \{(x, y, z) \in \mathbb{R}^3; xyz = 0\}$ (triple crossing); $\mathcal{C} = \{(x, y, z) \in \mathbb{R}^3; z^2 - x^2 - y^2 = 0\}$ (cone) or $\mathcal{W} = \{(x, y, z) \in \mathbb{R}^3; zx^2 - y^2 = 0\}$ (Whitney's umbrella).

Keywords: non-smooth dynamical system, singular perturbation, sliding vector field
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(Some figures may appear in colour only in the online journal)

