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Minimal sets of periods for Morse–Smale diffeomorphisms on non-orientable compact surfaces without boundary

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We study the minimal set of (Lefschetz) periods of the C^1 Morse–Smale diffeomorphisms on a non-orientable compact surface without boundary inside its class of homology. In fact our study extends to the C^1 diffeomorphisms on these surfaces having finitely many periodic orbits, all of them hyperbolic and with the same action on the homology as the Morse–Smale diffeomorphisms. We mainly have two kinds of results. First, we completely characterize the possible minimal sets of periods for the C^1 Morse–Smale diffeomorphisms on non-orientable compact surface without boundary of genus g with $1 \leq g \leq 9$. But the proof of these results provides an algorithm for characterizing the possible minimal sets of periods for the C^1 Morse–Smale diffeomorphisms on non-orientable compact surfaces without boundary of arbitrary genus. Second, we study what kind of subsets of positive integers can be minimal sets of periods of the C^1 Morse–Smale diffeomorphisms on a non-orientable compact surface without boundary.

Keywords: Morse–Smale diffeomorphism; Lefschetz number; zeta function; set of periods; minimal set of periods; non-orientable compact surfaces

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1. Introduction

In the study of the discrete dynamical systems, and in particular in the study of the orbits of self-maps defined on a given compact manifold, the periodic orbits play an important role. In these last 40 years, there were many results showing that some simple assumptions force qualitative and quantitative properties (like the set of periods) of a map. One of the first results in this direction was the paper *Period three implies chaos* for the interval continuous self-maps, see Ref. [19].

Probably for continuous self-maps on compact manifolds, the most useful tool for studying the existence of fixed and periodic points is the Lefschetz fixed point theorem and its improvements, see for instance [1,2,6–8,10,15,16,20,23]. The Lefschetz zeta function $\zeta_f(t)$ simplifies the study of the periodic points of f. It is a generating function for the Lefschetz numbers of all iterates of f. All these notions are defined in Section 3.

Here, we restrict our attention to the relevant class of discrete smooth dynamical systems given by the *Morse–Smale diffeomorphisms*. We recall some basic definitions and notation that will allow to define them.

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