A SURVEY ON THE MINIMAL SETS OF LEFSCHETZ PERIODS FOR MORSE–SMALE DIFFEOMORPHISMS ON SOME CLOSED MANIFOLDS

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Dedicated to Jorge Lewowicz in occasion of his Doctorado Honoris Causa *conferred by* Universidad de La República.

Abstract. We present the actual state of the study of the minimal sets of Lefschetz periods $\mathrm{MPer}_L(f)$ for the Morse–Smale diffeomorphisms on some closed manifolds, as the connected compact surfaces (orientable or not) without boundary, the n–dimensional torus and some other manifolds. The results on $\mathrm{MPer}_L(f)$ are valid for C^1 self–maps on the mentioned closed manifolds with finitely many periodic points all of them hyperbolic such that all the eigenvalues of the induced maps on homology are roots of unity. This class of maps includes the Morse–Smale diffeomorphisms.

1. Introduction

In the study of the discrete dynamical systems and, in particular in the study of the orbits of self–maps defined on a given compact manifold, the periodic orbits play an important role. These last forty years there was many results showing that some simple assumptions force qualitative and quantitative properties (like the set of periods) of a map. One of the first results in this direction was the famous paper *Period three implies chaos* for the interval continuous self–maps, see [24].

One of the most used tool for studying the existence of fixed points and periodic points, for continuous self maps on compact manifolds, and more generally topological spaces which are retract of finite simplicial complexes, is the Lefschetz fixed point theorem and its improvements (*cf.* [1, 2, 7, 8, 9, 11, 18, 19, 25, 30]). The Lefschetz zeta function $\zeta_f(t)$ simplifies the study of the periodic points of f. It is a generating function for the Lefschetz numbers of all iterates of f. All these notions are defined in Section 3.

The Morse-Smale diffeormorphisms have simple dynamic behaviour, however they are an important class of discrete dynamical systems. Our objective is to describe the periodic structure of these systems, in particular their minimal sets of periods. The results that we present here are valid for a class of maps that includes the Morse-Smale diffeomorphisms, i.e. C^1 maps having finitely many periodic points all of them hyperbolic and with the same action on the homology as the Morse–Smale diffeomorphisms.

Many papers have been published analyzing the relationships between the dynamics of the Morse–Smale diffeomorphisms and the topology of the manifold where they are defined, see for instance [3, 4, 5, 11, 12, 13, 14, 15, 31, 33, 35, 36, 37]. The Morse–Smale diffeomorphisms have a relatively simple orbit structure. In fact, their set of periodic orbits is finite, and their structure is preserved under small C^1 perturbations.

Date: August 21, 2013.

2010 *Mathematics Subject Classification.* 37D15, 37E15.

Key words and phrases. Morse-Smale diffeomorphisms, quasi-nilpotent maps, Lefschetz numbers, zeta functions, set of periods, minimal set of periods.