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Partially periodic point free self-maps on surfaces, graphs, wedge sums and products of spheres

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Let (X,f) be a topological discrete dynamical system. We say that it is partially periodic point free up to period n, if f does not have periodic points of periods smaller than $n+1$. When X is a compact connected surface, a connected compact graph, or $\mathbb{S}^{2m} \vee \mathbb{S}^m \vee \cdots \vee \mathbb{S}^m$, we give conditions on X, so that there exist partially periodic point free maps up to period n. We also introduce the notion of a Lefschetz partially periodic point free map up to period n. This is a weaker concept than partially periodic point free up to period n. We characterize the Lefschetz partially periodic point free self-maps for the manifolds $\mathbb{S}^n \times \overset{k}{\cdots} \times \mathbb{S}^n$, $\mathbb{S}^n \times \mathbb{S}^m$ with $n \neq m$, $\mathbb{C}P^n$, $\mathbb{H}P^n$ and $\mathbb{O}P^n$.

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1. Introduction

Let X be a topological space and let $f : X \to X$ be a continuous map. A (*discrete*) *topological dynamical system* is formed by the pair (X,f).

We say that $x \in X$ is a *periodic point of period* k if $f^k(x) = x$ and $f^j(x) \neq x$ for $j = 1, \ldots, k-1$. We denote by $\mathrm{Per}(f)$ the set of all periods of f.

The set $\{x, f(x), f^2(x), \ldots, f^n(x), \ldots\}$ is called the *orbit* of the point $x \in X$. To study *the dynamics of a map f* is to study all the different kinds of orbits of f. If x is a periodic point of f of period k, then its *orbit* is $\{x, f(x), f^2(x), \ldots, f^{k-1}(x)\}$, and it is called a *periodic orbit*.

Often the periodic orbits play an important role in the dynamics of a discrete dynamical system, and for studying them we can use topological tools. One of the best-known results in this direction is the result contained in the well-known paper entitled '*Period three implies chaos*' for continuous self-maps on the interval, see [19].

If $\mathrm{Per}(f) = \emptyset$ then we say that the map f is *periodic point free*. There are several papers studying different classes of periodic point free self-maps on the annulus, see [12,16], or on the two-dimensional torus, see [2,14,18].

If $\mathrm{Per}(f) \cap \{1, 2, \ldots, n\} = \emptyset$ then we say that the map f is *partially periodic point free up to period n*. If $n = 1$, we say that f is *fixed point free*. Different classes of partially periodic point free self-maps are studied in [6,25,28].

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