C^1 SELF-MAPS ON CLOSED MANIFOLDS WITH FINITELY MANY PERIODIC POINTS ALL OF THEM HYPERBOLIC

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Abstract. Let X be a connected closed manifold and f a self-map on X. We say that f is almost quasi-unipotent if every eigenvalue λ of the map f_{*k} (the induced map on the k-th homology group of X) which is neither a root of unity, nor a zero, satisfies that the sum of the multiplicities of λ as eigenvalue of all the maps f_{*k} with k odd is equal to the sum of the multiplicities of λ as eigenvalue of all the maps f_{*k} with k even.

We prove that if f is C^1 having finitely many periodic points all of them hyperbolic, then f is almost quasi-unipotent.

Keywords: hyperbolic periodic point; differentiable map; Lefschetz number; Lefschetz zeta function; quasi-unipotent map; almost quasi-unipotent map

MSC 2010: 37C05, 37C25, 37C30

1. Introduction and statement of the main result

Let X be a topological space and $f\colon X \to X$ a continuous map on X. We say that $x \in X$ is a *periodic point of period* p if $f^p(x) = x$ and $f^j(x) \neq x$ for $1 \leqslant j \leqslant p-1$.

Let X be a differentiable manifold and f a differentiable map. We say that a periodic point of period p is *hyperbolic* if the derivative of f^p at x, i.e. $Df^p_x\colon TX_x \to TX_x$, has no eigenvalues of modulus equal to 1.

If the dimension of X is n, the map f induces a homomorphism on the k-th rational homology group of X for $0 \leqslant k \leqslant n$, i.e. $f_{*k}\colon H_k(X, \mathbb{Q}) \to H_k(X, \mathbb{Q})$. Here

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