International Journal of Bifurcation and Chaos, Vol. 27, No. 3 (2017) 1750039 (6 pages)

DOI: 10.1142/S0218127417500390

Periodic Orbits of the Planar Anisotropic Kepler Problem

Elbaz I. Abouelmagd
Celestial Mechanics Unit, Astronomy Department, National Research Institute of Astronomy and Geophysics (NRIAG), Helwan, Cairo, Egypt
eabouelmagd@gmail.com
eabouelmagd@nriag.sci.eg

Jaume Llibre
Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain
jllibre@mat.uab.cat

Juan Luis García Guirao
Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Hospital de Marina, 30203-Cartagena, Región de Murcia, Spain
juan.garcia@upct.es

Received September 13, 2016; Revised December 31, 2016

In this paper, we prove that at every energy level the anisotropic Kepler problem with small anisotropy has two periodic orbits which bifurcate from elliptic orbits of the Kepler problem with high eccentricity. Moreover we provide approximate analytic expressions for these periodic orbits. The tool for proving this result is the averaging theory.

Keywords: Anisotropic Kepler problem; periodic orbits; averaging theory.

1. Introduction

The anisotropic Kepler problem is a modified model of the Kepler problem. This model can be used to describe the motion of two-body in an anisotropic configuration plane under a mutual gravitational attraction described by Newton's universal law of gravitation.

The anisotropic Kepler problem comes originally from quantum mechanics, in this model the flat space is replaced with an anisotropic one, it was introduced by Gutzwiller [1981, 1973, 1977, 1971], and later studied by other authors, see for instance Devaney [1978, 1981, 1982], Casasayas and Llibre [1984], and Vidal [2001].

Gutzwiller in his researches wanted to find an approximation of the quantum mechanical energy levels for a chaotic system. He chose to study the anisotropic Kepler problem because it is a chaotic system and it is considered one of the most suitable models to study physical phenomena found in semiconductors.