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On topological entropy, Lefschetz numbers and Lefschetz zeta functions

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ABSTRACT

In the present article we give sufficient conditions for C^∞ self-maps on some connected compact manifolds in order to have positive entropy. The conditions are given in terms of the Lefschetz numbers of the iterates of the map and/or its Lefschetz zeta function. We consider the cases where the manifold is a compact orientable and non-orientable surface, the n-dimensional torus, the product of n spheres of dimension ℓ and the product of spheres of different dimensions.

1. Introduction

The topological entropy of a topological dynamical system is a nonnegative real number which measures the complexity of the system. Topological entropy was first introduced in 1965 by Adler, Konheim and McAndrew [1]. Later on Dinaburg [8] and Bowen [7] provided a distinct definition inspired by the Hausdorff dimension. For a system defined by the iteration of a function, the topological entropy is the exponential growth rate of the number of distinguishable orbits of the iterates.

Here we provide sufficient conditions in order that the topological entropy of a map in a compact manifold be positive in terms of the Lefschetz numbers of the iterates of the map or in function of the Lefschetz zeta function of the map.

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