STRUCTURAL STABILITY OF CONSTRAINED POLYNOMIAL SYSTEMS

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Abstract

The structural stability of constrained polynomial differential systems of the form $a(x,y)x'+b(x,y)y' = f(x,y)$, $c(x,y)x' + d(x,y)y' = g(x,y)$, under small perturbations of the coefficients of the polynomial functions a, b, c, d, f and g is studied. These systems differ from ordinary differential equations at 'impasse points' defined by $ad - bc = 0$. Extensions to this case of results for smooth constrained differential systems [**7**] and for ordinary polynomial differential systems [**5**] are achieved here.

1. *Introduction and statement of the main results*

This paper is concerned with the orbit structure and stability properties of planar differential systems of the form

$$\begin{aligned} a(x,y)x' + b(x,y)y' &= f(x,y), \\ c(x,y)x' + d(x,y)y' &= g(x,y), \end{aligned} \tag{1}$$

where $a(x,y)$, $b(x,y)$, $c(x,y)$ and $d(x,y)$ are polynomials of degree m, while $f(x,y)$ and $g(x,y)$ are polynomials of degree n. They will be called *constrained polynomial systems* of degree (m,n).

Constrained smooth systems (that is, systems where the coefficients a, b, c, d, f and g are of class C^r), have been studied in [**7**] and [**8**]. Sources of applications for these systems can be found in [**2**].

In matrix notation, system (1) can be written as

$$A(\mathbf{x})\mathbf{x}' = F(\mathbf{x}), \tag{2}$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad A(\mathbf{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}(\mathbf{x}), \quad F(\mathbf{x}) = \begin{pmatrix} f \\ g \end{pmatrix}(\mathbf{x}), \quad (\cdot)' = \frac{d(\cdot)}{dt}.$$

A system of the form (2) will also be referred to as (A,F). The *impasse singularities* of (2) are those points $\mathscr{I}_A$ where $\delta_A = \det(A)$, the determinant of the matrix A, vanishes.

The *solutions*, *orbits*, *singular points*, etc. of system (2) are defined only outside the impasse singularities by the corresponding similar elements of the system

$$\mathbf{x}' = A^{-1}(\mathbf{x})F(\mathbf{x}), \quad \mathbf{x} \in \mathbf{R}^2 \setminus \mathscr{I}_A. \tag{3}$$

System (2) differs from the ordinary differential system given by (3) only on $\mathscr{I}_A$.

Received 3 March 1997; revised 9 September 1997.

1991 *Mathematics Subject Classification* 34C35, 34D30.

The first author was supported by a Fapesp grant number 96/1907–2 and by a CIRIT grant number 1996BEAI400252 during the preparation of this work. The second author was partially supported by a Fapesp grant number 96/3450–0.