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Dynamic systems behaviour analysis and design based on the qualitative theory of differential equations: the Boost power converter case

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This paper uses the qualitative theory of differential equations to analyse/design the dynamic behaviour of control systems. In particular, the Poincaré compactification and the Poincaré–Hopf theorem are used for analysing the local dynamics near the finite and infinite equilibrium points. As an application, a large signal characterisation of a Boost type power converter in closed loop, including its equilibrium/bifurcation points and its global dynamics, which depends upon the value of the load resistance, is studied.

Keywords: power converters; nonlinear systems; control theory; Poincaré compactification; topological index

1. Introduction

In general, after selecting a controller for a dynamic system, it is desirable to have a globally stable closed-loop behaviour. The Lyapunov approach is commonly invoked to demonstrate the stability of a control system, see for instance Khalil (2000). Under this approach, if $x = 0$ is an equilibrium point for $\dot{x} = f(x)$ and $D \subset \mathbb{R}^2$, then given a continuously differentiable candidate function $V : D \to \mathbb{R}$, such that $\dot{V} < 0$ along the solutions of $\dot{x} = f(x)$ in $D\backslash\{0\}$ then $x = 0$ is asymptotically stable. That is, all trajectories of $\dot{x} = f(x)$ contained in D converge to the origin $x = 0$.

For a dynamic linear control system, this technique explains if such a system is stable or not. For a nonlinear system, however, the procedure for obtaining the candidate function is difficult, implies trial and error, and in many cases no such function exists. A simple example is a nonlinear differential equation with two or more equilibrium points. Under these circumstances, it is not possible to resort to using the Lyapunov approach to analyse the global dynamics of the system and defining the boundary of the attraction zone of a locally stable equilibrium point is a major control problem. In view of the above, the use of the qualitative theory of differential equations is emphasised in order to achieve this purpose. In particular, the Poincaré compactification method for studying the dynamics of the system near infinity will be considered, and together with the Poincaré–Hopf theorem the local dynamics at the equilibrium points, finite and infinite, will be analysed.

The Poincaré compactification method introduced by Poincaré (1881) has been referenced for instance in Andronov, Leontovich, Gordon, and Maier (1973) and Dumortier, Llibre, and Artés (2006). This method was used in Dickson and Perko (1970) to characterise the phase portrait of bounded quadratic systems.

A fast introduction to the Poincaré compactification is let $\mathbb{S}^2$ be the 2-dimensional sphere of radius one centred at the origin of coordinates $(x, y, z) = (0, 0, 0)$ of $\mathbb{R}^3$. The Poincaré compactification method consists in doing two copies of a polynomial differential system flow in $\mathbb{R}^2$, which is identified with the tangent plane to the sphere $\mathbb{S}^2$ at its north pole (0, 0, 1).

These two copies are obtained through central projections of this tangent plane, one to the open northern hemisphere of $\mathbb{S}^2$ and another to the open southern hemisphere. Let $\mathbb{S}^1$ be the equator of $\mathbb{S}^2$. The flow defined in $\mathbb{S}^2 \setminus \mathbb{S}^1$ is extended to the equator $\mathbb{S}^1$, which corresponds to the infinity of the tangent plane $\mathbb{R}^2$. Finally, using the projection $(x, y, z) \to (x, y)$ the closed northern hemisphere is equivalent to a closed unit disc, called the Poincaré disc. This extension allows studying the flow of a polynomial differential system in a neighbourhood of the infinity. The plane and the infinity of $\mathbb{R}^2$ are identified with the interior of the Poincaré disc and with the boundary disc $\mathbb{S}^1$, respectively. This technique will be used to explain the behaviour of a Boost power converter near the infinity.

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