J. Differential Equations 250 (2011) 1747–1766

Contents lists available at ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

Comet- and Hill-type periodic orbits in restricted $(N+1)$-body problems

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ARTICLE INFO

Article history:
Received 13 July 2010
Available online 6 September 2010

ABSTRACT

We consider the planar restricted $(N+1)$-body problem where the interaction potential between the particle and the primaries is taken to be a finite sum of terms of the form $(\text{distance})^{-\alpha}$ with $\alpha > 0$.

The primaries are assumed to be in a relative equilibrium, that is, they form a uniformly rotating rigid configuration.

We show two results. First, if the infinitesimal particle is far from the primaries and the long range dominant term γ/r^{α} of the potential is such that $\gamma < 0$ and $\alpha \neq 2$, then there exist two one-parameter families of large nearly circular periodic solutions. These solutions, called *comet solutions*, are elliptic and KAM stable for $\alpha < 2$, and unstable for $\alpha > 2$. Second, if the infinitesimal particle is close to one of the primaries and the short range dominant term γ/r^{α} of the potential near that primary is such that $\gamma < 0$ and $\alpha \neq 2$, then there exist two one-parameter families of nearly circular periodic solutions, called *Hill solutions*, that encircle the nearby primary. For $\alpha > 2$, these orbits are unstable.

The methodology applied involves appropriate symplectic scalings, Poincaré's continuation method and averaging theory. The KAM stability of the comet periodic orbits is decided by verifying Arnol'd's non-degeneracy conditions.

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0022-0396/$ – see front matter
doi:10.1016/j.jde.2010.08.005