POLYNOMIAL AND RATIONAL INTEGRABILITY OF POLYNOMIAL HAMILTONIAN SYSTEMS

JAUME LLIBRE[1], CRISTINA STOICA[2] AND CLÀUDIA VALLS[3]

ABSTRACT. We study the relationship between the existence of Darboux polynomials and additional polynomial or rational first integrals for polynomial Hamiltonian systems satisfying certain symmetries.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

One of the most natural class of canonical Hamiltonian systems which appear in Mechanics is given by Hamiltonians expressed as the a sum of the kinetic and potential energy. Maciejewski, Nakagawa and Przybylska in [6], Maciejewski and Przybylska in [5], and and later on Garcia, Grau and Llibre in [2] studied the integrability of canonical Hamiltonian systems in $\mathbb{C}^{2m}$ where the Hamiltonian is given by a polynomial expression of the form

$$H(q,p) = \frac{1}{2}\sum_{i=1}^{m} \mu_i p_i^2 + V(q), \tag{1}$$

where $q, p \in \mathbb{C}^{2m}$, the potential energy $V(q)$ is a polynomial, and $\mu_i \in \mathbb{C}$ for $i = 1, \ldots, m$.

In this paper, first we extend results on Hamiltonians of the form (1) as obtained in the aforementioned papers to time reversible Hamiltonian systems in $\mathbb{C}^{2m}$ with an arbitrary polynomial Hamiltonian $H(q,p)$. For such systems, under convenient assumptions, we deduce the existence of a second polynomial first integral independent of the Hamiltonian.

Second, we consider invariant polynomial Hamiltonian systems in $\mathbb{C}^{2m}$ under an involution acting on $(q, p, H(q,p))$. In this case, provided some additional assumptions are satisfied, we obtain a second polynomial or rational first integral independent of the Hamiltonian.

A canonical Hamiltonian system defined on $\mathbb{C}^{2m}$ with m degrees of freedom and Hamiltonian $H(q,p)$ is given by

$$\frac{dq_i}{dt} = \frac{\partial H(q,p)}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H(q,p)}{\partial q_i}, \quad \text{for } i = 1, \ldots, m, \tag{2}$$

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