

Relationships between limit cycles and algebraic invariant curves for quadratic systems

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Abstract

Algebraic limit cycles for quadratic systems started to be studied in 1958. Up to now we know 7 families of quadratic systems having algebraic limit cycles of degree 2, 4, 5 and 6. Here we present some new results on the limit cycles and algebraic limit cycles of quadratic systems. These results provide sometimes necessary conditions and other times sufficient conditions on the cofactor of the invariant algebraic curve for the existence or nonexistence of limit cycles or algebraic limit cycles. In particular, it follows from them that for all known examples of algebraic limit cycles for quadratic systems those cycles are unique limit cycles of the system.

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1. Introduction and statement of the main results

We shall study polynomial vector fields in \mathbb{R}^2 defined by systems

$$\dot{x} = p(x, y), \quad \dot{y} = q(x, y), \quad (1)$$

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