

Classification of quadratic systems admitting the existence of an algebraic limit cycle

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Abstract

In the paper we find a set of necessary conditions that must be satisfied by a quadratic system in order to have an algebraic limit cycle. We find a countable set of ≤ 5 parameter families of quadratic systems such that every quadratic system with an algebraic limit cycle must, after a change of variables, belong to one of those families. We provide a classification of all the quadratic systems which can have an algebraic limit cycle based on geometrical properties of the embedding of the system in the Poincaré compactification of \mathbb{R}^2 . We propose names for all the classes we distinguish and we classify all known examples of quadratic systems with algebraic limit cycle. We also prove the integrability of certain classes of quadratic systems. © 2006 Elsevier Masson SAS. All rights reserved.

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1. Introduction and statement of the main results

Since Darboux [12] has found in 1878 connections between algebraic geometry and the existence of first integrals of polynomial differential systems (polynomial planar vector fields), algebraic invariant curves are a central object in the theory of integrability of polynomial systems in \mathbb{R}^2 . Today, after more than one century of investigations the theory of invariant algebraic

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