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ON THE LIMIT CYCLES OF POLYNOMIAL VECTOR FIELDS

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Abstract. In this paper we study the limit cycles which can bifurcate from the periodic orbits of the center located at the origin of the quadratic polynomial differential system $\dot{x} = -y(1+x)$, $\dot{y} = x(1+x)$, and of the cubic polynomial differential system $\dot{x} = -y(1-x^2-y^2)$, $\dot{y} = x(1-x^2-y^2)$, when we perturb them in the class of all polynomial vector fields with quadratic and cubic homogenous nonlinearities, respectively. For doing this study we use the averaging theory.

Keywords. Limit cycle; Periodic orbit; Center; Reversible center; Averaging method.

AMS (MOS) subject classification: 34C29; 34C25; 47H11.

1 Introduction and statement of the results

After the definition of limit cycle due to Poincaré [14], the statement of the 16–th Hilbert's problem [9], the discover that the limit cycles are important in the nature by Liénard [11], ... the study of the limit cycles of the planar differential systems has been one of the main problems of the qualitative theory of the differential equations.

One of the best ways of producing limit cycles is by perturbing the periodic orbits of a center. This has been studied intensively perturbing the periodic orbits of the centers of the quadratic polynomial differential systems see the book of Christopher and Li [6], and the references quoted there.

It is well known that if a quadratic polynomial differential system has a limit cycles this must surround a focus. Up to know the maximum number of known limit cycles surrounding a focus of a quadratic polynomial differential system is 3, which coincides with the maximum number of small limit cycles which can bifurcate by Hopf from a singular point of a quadratic polynomial differential system, see Bautin [1]. But as far as we know up to now there are few quadratic centers for which it is proved that the perturbation of their periodic orbits inside the class of all quadratic polynomial differential systems can produce 3 limit cycles. These are the center whose exterior boundary