HOPF BIFURCATION OF A DELAY DIFFERENTIAL EQUATION WITH TWO DELAYS

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Dedicated to Professor Dr. Ioan A. Rus on his 70th birthday.

Abstract. We consider a delay differential equation with two delays. The Hopf bifurcation of this equation is investigated together with the stability of the bifurcated periodic solution, its period and the bifurcation direction. Finally, three applications are given.

1. Introduction

Hopf [9] was the first who state a theorem concerning the bifurcation of periodic solutions from a singular point of an ordinary differential equation. Many generalizations to infinite–dimensional systems have been given (see [12] for references). As far as we know, the first statement similar to this theorem for retarded functional–differential equations was given by Chow and Mallet–Paret in a course at Brown University in 1974, see [7]. Efficient procedures for determining the stability and the amplitude of the bifurcating periodic orbit using a method of averaging have been given by Chow and Mallet–Paret [4]. The global existence of a Hopf bifurcation as a function of initial data and the period has been discussed by Chow and Mallet–Paret [5] and Nussbaum [14]. The interest on the periodic orbits of a delay differential equation has increased strongly these last years, see for instance [2], [3], [13], [15]–[17].

In this paper we study the delay differential equation of the form

$$\begin{aligned} \dot{x}(t) &= -(a\pi+\mu)[x(t-1)+x(t-2)+G_2(x(t),x(t-1),x(t-2))]\cdot \\ &\quad [1+G_1(x(t),x(t-1),x(t-2))], \end{aligned} \tag{1}$$

where $9/100 \le a \le \sqrt{3}/9$, and $G_1(x,y,z)$ and $G_2(x,y,z)$ are analytic functions in a neighborhood of $\mathbf{0} \in \mathbb{R}^3$, starting with terms of degree at least 1 and 2 respectively. We prove that equation (1) exhibits a Hopf bifurcation and we discuss for distinct functions G_1 and G_2 about the period, the

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