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Periodic Orbits for Some Systems of Delay Differential Equations

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Abstract We provide sufficient conditions for the existence of periodic orbits of some systems of delay differential equations with a unique delay. We extend Kaplan–Yorke's method for finding periodic orbits from a delay differential equation with several delays to a system of delay differential equations with a unique delay.

Keywords delay differential system, periodic orbit

MR(2000) Subject Classification 34K13, 34K18

1 Introduction and Statement of the Main Results

Many papers dedicated to studying the periodic orbits of a delay differential equation using Kaplan–Yorke's method have been published these last few years, see for instance [1–6], [8–12]. In this paper we extend Kaplan–Yorke's method for finding periodic orbits from a delay differential equation with several delays to a system of delay differential equations with a unique delay. As far as we know this method has never been used before for finding periodic orbits for a system of delay differential equations with more than one equation.

Here we study the existence of periodic orbits for the following delay differential systems

$$\begin{aligned} \dot{x}(t) &= -F(x(t-s), y(t-s), x(t)), \\ \dot{y}(t) &= F(y(t), x(t-s), y(t-s)); \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}(t) &= -F(x(t-s), y(t-s), -x(t-s)), \\ \dot{y}(t) &= F(-y(t-s), x(t-s), y(t-s)); \end{aligned} \tag*{(1)$'$}$$

$$\begin{aligned} \dot{x}(t) &= F(x(t-s), y(t-s), z(t-s)), \\ \dot{y}(t) &= F(y(t-s), z(t-s), -x(t-s)), \\ \dot{z}(t) &= F(-z(t-s), x(t-s), y(t-s)); \end{aligned} \tag{2}$$

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