DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Volume **3**, Number **2**, April **1997** pp. **235–241**

REGULARIZATION OF DISCONTINUOUS VECTOR FIELDS IN DIMENSION THREE

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(Communicated by A. Bressan)

Abstract. In this paper vector fields around the origin in dimension three which are approximations of discontinuous ones are studied. In a former work of Sotomayor and Teixeira [6] it is shown, via regularization, that Filippov's conditions are the natural ones to extend the orbit solutions through the discontinuity set for vector fields in dimension two. In this paper we show that this is also the case for discontinuous vector fields in dimension three. Moreover, we analyse the qualitative dynamics of the local flow in a neighborhood of the codimension zero regular and singular points of the discontinuity surface.

1. **Introduction.** Let $f : \Re^3, 0 \to \Re, 0$ be a germ of a C^∞ function having 0 as regular value. Call $S = \{f^{-1}(0)\}$. Denote by χ^r the space of C^r vector fields on $\Re^3, 0$ with $r > 1$, and by χ the space of Lipschitz vector fields on $\Re^3, 0$.

Let Ω^r be the space of vector fields Z on $\Re^3, 0$ defined by

$$Z(q) = \begin{cases} X(q) & \text{if} \quad f(q) > 0, \\ Y(q) & \text{if} \quad f(q) < 0, \end{cases}$$

where X,Y $\in \chi^r$. This vector field is denoted by Z = (X,Y). The study of such systems arises in applications to mechanics [10] , control theory [4] and economics [3] and [2].

Let $Z = (X, Y)$ be in Ω^r. Given a positive number ε we consider the *piecewise linear* function $\varphi_\varepsilon : \Re \to \Re$ defined by

$$\varphi_\varepsilon(t) = \begin{cases} 0 & \text{if} \quad t \le -\varepsilon, \\ \frac{t+\varepsilon}{2\varepsilon} & \text{if} \quad t \in (-\varepsilon, \varepsilon), \\ 1 & \text{if} \quad t \ge \varepsilon. \end{cases}$$

An ε–regularization of $Z = (X, Y) \in \Omega^r$ is a vector field Z_ε in χ defined by

$$Z_\varepsilon(q) = [1 - \varphi_\varepsilon(f(q))]Y(q) + \varphi_\varepsilon(f(q))X(q).$$

1991 *Mathematics Subject Classification.* 58F09, 34C35.
Key words and phrases. Regularization and discontinuous vector fields.