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Global asymptotic stability for a class of discontinuous vector fields in $\mathbb{R}^2$

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Consider a partition of the plane $\mathbb{R}^2$ by squares of the same size. For the class of discontinuous vector fields formed by infinitely many linear vector fields on every square of the above partition, we provide sufficient conditions for the existence of a global asymptotic equilibrium point. Such vector fields come from control theory.

1. Introduction and statement of the main result

A matrix is *Hurwitz* if all its eigenvalues have negative real parts. A differential system

$$x'(t) = F(x(t)), \tag{1}$$

where $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ is C^1, is Hurwitz if its Jacobian matrix $DF(x)$ is Hurwitz at every point $x \in \mathbb{R}^2$.

Suppose that the origin of $\mathbb{R}^2$ is an equilibrium point of a Hurwitz system (1). Then, by the Hartman–Grobman Theorem [1] the origin is a local asymptotically stable solution. The question is what hypothesis we have to add to $F(x)$ to assure that the origin is a global attractor. In 1960, Markus and Yamabe [2] conjectured as follows: *Let $x' = F(x)$ be a differential system, with F of class C^1. If it is Hurwitz and the origin is the unique equilibrium point, then the origin is a global asymptotically stable solution*. More than thirty years later this conjecture was proved independently by three authors, Fessler [3], Glutsyuk [4] and Gutiérrez [5].

We consider the sets

$$S = \left\{(x_1, x_2) \in \mathbb{R}^2 : x_1 = \frac{n}{2} \text{ or } x_2 = \frac{n}{2} \text{ for some } n \in \mathbb{Z}\right\},$$

$$S'' = \left\{(x_1, x_2) \in \mathbb{R}^2 : x_1 = \frac{m}{2} \text{ and } x_2 = \frac{n}{2} \text{ for some } m, n \in \mathbb{Z}\right\},$$

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