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On the periodic orbits of the fourth-order differential equation $u'''' + qu'' - u = \varepsilon F(u, u', u'', u''')$

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ABSTRACT

We provide sufficient conditions for the existence of periodic solutions of the fourth-order differential equation

$$u'''' + qu'' - u = \varepsilon F\big(u, u', u'', u'''\big),$$

where q and ε are real parameters, ε is small and F is a nonlinear function.

1. Introduction and statement of the main results

The objective of this paper is to study the periodic solutions of the fourth-order differential equation

$$u'''' + qu'' - u = \varepsilon F\big(u, u', u'', u'''\big), \tag{1}$$

where q and ε are real parameters, ε is small and F is a nonlinear function. The prime denotes derivative with respect to an independent variable l.

Equations of the form (1) appear in many contexts, we only mention some of them in what follows. For instance, Champneys [8] analyzes a class of Eqs. (1) looking mainly for homoclinic orbits.

When $F = \pm u^2$ Eq. (1) can come from the description of the travelling-wave solutions of the Korteweg–de Vries equation with an additional fifth-order dispersive term. This equation has been used to describe chains of coupled nonlinear oscillators [19] and most notably gravity-capillary shallow water waves [3,13,22]. Extended fifth-order Korteweg–de Vries equations have been considered in [6,9,10,15,16,18].

Other derivation of (1) with $F = \pm u^2$ appears in describing the displacement u of a compressed strut with bending softness resting on a nonlinear elastic foundation with dimensionless restoring force proportional to $u - u^2$ [11,12].

Another nonlinearity is $F = \pm u^3$, then Eq. (1) is called the *Extended Fischer–Kolmogorov* equation or the *Swift–Hohenberg* equation. Eq. (1) with this nonlinearity appears when we study travelling-wave solutions of the nonlinear Schrödinger equation with an additional fourth-order dispersion term [5,14] and in other places, see for instance the book [20] and [2,7].

We recall that a simple zero r_0^* of a real function $\mathcal{F}(r_0)$ is defined by $\mathcal{F}(r_0^*) = 0$ and $(d\mathcal{F}/dr_0)(r_0^*) \neq 0$.

Our main result on the periodic solutions of the fourth-order differential equation (1) is the following one.

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