PERIODIC SOLUTIONS OF DISCONTINUOUS SECOND ORDER DIFFERENTIAL SYSTEMS

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ABSTRACT. We provide sufficient conditions for the existence of periodic solutions of some classes of autonomous and non-autonomous second order differential equations with discontinuous right-hand sides. In the plane the discontinuities considered are given by the straight lines either x = 0, or xy = 0. Two applications of these results are made, one to control systems with variable structure, and the other to small external periodic excitation of a discontinuous nonlinear oscillator.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

In these last tens the study of discontinuous differential systems became relevant in the boundary between Mathematics, Physics and Engineering. In the book [2] and in the survey [10] there are different models coming from the impacting motion in mechanical systems, or from switchings in electronic systems, or from hybrid dynamics in control systems, and so on. All of these models are formulated with differential equations with discontinuous right-hand sides. Also, many studies have been done in the qualitative aspects of the phase space of discontinuous differential systems, see for instance the hundreds of references quoted in [2] and [10].

In this paper we are mainly interested in the study of the periodic solutions of autonomous and non–autonomous second order differential equations with discontinuous right–hand sides. Recently discontinuous second order differential equations have been studied for several authors, mainly non–autonomous ones. Thus, discontinuous differential equations of the form

$$u'' + u + \alpha \operatorname{sign}(y) = F(\theta),$$

where F is a periodic function has been studied in [7]. In [5] periodic solutions of discontinuous differential equations of the form $u'' + G(u) = F(\theta)$ are analyzed, where F is periodic and continuous, and G is continuous except at u = 0. In [6] the authors studied the periodic solutions of the discontinuous differential equations $u'' + \eta \operatorname{sign}(u) = \alpha \operatorname{sin}(\beta t)$.

Our main results will provide sufficient conditions for the existence of periodic solutions of the following two classes of autonomous second order differential equations with discontinuous right–hand sides:

(1)
$$u'' + u + \varepsilon \alpha \operatorname{sign}(u)G(u, u') = \varepsilon H(u, u')$$

(2)
$$u'' + u + \varepsilon \alpha \operatorname{sign}(uu')G(u, u') = \varepsilon H(u, u')$$

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