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Piecewise linear differential systems without equilibria produce limit cycles?

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Abstract In this article, we study the planar piecewise differential systems formed by two linear differential systems separated by a straight line, such that both linear differential have no equilibria, neither real nor virtual. When the piecewise differential system is continuous, we show that the system has no limit cycles. But when the piecewise differential system is discontinuous, we show that it can have at most one limit cycle.

Keywords Limit cycles · Continuous piecewise linear differential systems · Discontinuous piecewise differential systems

Mathematics Subject Classification 34C29 · 34C25 · 47H11

1 Introduction and statement of the main result

The study of the so-called discontinuous linear differential systems goes back to Andronov et al. [1] and has received special attention from the mathematical community nowadays, mainly because these systems are widely used to model processes appearing in electronics, mechanics, economy, etc. See, for instance, the books of di Bernardo et al. [5] and Simpson [28], the survey of Makarenkov and Lamb [26], and the hundreds of references which appear in these last three cited works.

The simplest possible piecewise linear differential systems are the ones formed by two linear differential systems separated by a straight line. We note that for these apparently simple systems, when they are continuous, some serious work is necessary for proving that they have at most one limit cycle, see [7] and [21]. This solved the conjecture of Lum and Chua [25] done in 1990 that such continuous differential systems can have at most one limit cycle.

The study of the maximum number of limit cycles of these systems, when they are discontinuous, still is an open question. Up to now, we know that there are discontinuous systems with at least three limit cycles, see, for instance, [2–4,6,8–15,17–20,22,22,24].

We consider planar piecewise differential systems formed by two linear differential systems separated by the straight line $x = 0$, such that both linear differential have no equilibria, neither real nor virtual. We say that an equilibrium point (x_0, y_0) of the linear differential system defined in the half-plane $x \geq 0$ is *real* if it $x_0 \geq 0$, otherwise it is called *virtual*.

Theorem 1 *Continuous planar piecewise differential systems formed by two linear differential systems sep-*

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