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Limit cycles in Filippov systems having a circle as switching manifold

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ABSTRACT

It is known that planar discontinuous piecewise linear differential systems separated by a straight line have no limit cycles when both linear differential systems are centers. Here, we study the limit cycles of the planar discontinuous piecewise linear differential systems separated by a circle when both linear differential systems are centers. Our main results show that such discontinuous piecewise differential systems can have zero, one, two, or three limit cycles, but no more limit cycles than three.

Over the last few years, there has been a great deal of interest in studying the limit cycles of discontinuous piecewise differential systems formed by pieces of linear differential systems, mainly due to the applications of these differential systems in many natural problems; see, for more details, Sec. I. Up to now, the major part of these studies was done for the discontinuous piecewise differential systems formed by two pieces of linear differential systems separated by one straight line. Here, we study the limit cycles of the discontinuous piecewise differential systems formed by two pieces of linear differential centers separated by a circle. We prove that these differential systems can have at most three limit cycles and that there are differential systems having exactly zero, one, two, and three limit cycles. Therefore, in particular, we have solved the extension of the 16th Hilbert problem to this class of differential systems.

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

A. Historical facts

The problem of existence of limit cycles has been extensively treated in the literature since the early days of celestial mechanics. More recently, much work has been done on the rigorous mathematical foundation of nonsmooth dynamical systems problems, in particular, in the search for typical minimal sets for which there are no counterparts in the smooth universe. It is worth mentioning that some existing smooth techniques are useful in solving many nonsmooth problems.

Some of the orbits of planar differential systems are difficult to study, for example, the case of the limit cycles. Recall that a *limit cycle* of a differential system (S) is a periodic solution of (S), which is isolated in the set of all periodic solutions of (S). Concerning the nonsmooth universe and in the two-dimensional case, one can find many results on the existence of limit cycles when the switching set is an embedded curve in $\mathbb{R}^2$; see Refs. 1–15 and 18–29.

One of the main properties of smooth integrable systems in the plane $\mathbb{R}^2$ is that their periodic orbits usually appear in continuous one–parameter families, in contrast to the periodic orbits of piecewise nonsmooth integrable systems, which typically can be limit cycles; see Refs. 2–6 and 18–28.

Andronov *et al.*[1] began the study of the discontinuous piecewise linear differential systems in the plane, mainly motivated by their applications to some mechanical problems. Recently, the interest to this kind of differential systems increased due mainly to the fact that these differential systems model many processes that appear in mechanics, electronics, economy, etc. For these applications, see the survey of Makarenkov and Lamb[27] and the books of Simpson[30] and of di Bernardo *et al.*,[8] together with hundreds of references quoted in these works.