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LOWER BOUNDS FOR THE MAXIMUM NUMBER OF LIMIT CYCLES OF DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH A STRAIGHT LINE OF SEPARATION

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In this paper, we provide a lower bound for the maximum number of limit cycles of planar discontinuous piecewise linear differential systems defined in two half-planes separated by a straight line. Here, we only consider nonsliding limit cycles. For those systems, the interior of any limit cycle only contains a unique equilibrium point or a unique sliding segment. Moreover, the linear differential systems that we consider in every half-plane can have either a focus (F), or a node (N), or a saddle (S), these equilibrium points can be real or virtual. Then, we can consider six kinds of planar discontinuous piecewise linear differential systems: FF, FN, FS, NN, NS, SS. We provide for each of these types of discontinuous differential systems examples with two limit cycles.

Keywords: Nonsmooth differential system; limit cycle; piecewise linear differential system.

1. Introduction and Statement of the Main Results

The study of piecewise linear differential systems goes back to Andronov and coworkers [Andronov *et al.*, 1966], and today are still being studied by many researchers. Thus, these last years, considerable interest from the mathematical community is seen in trying to understand their dynamical richness, because such systems are widely used to model many real processes and different modern devices, see for more details [di Bernardo *et al.*, 2008] and the references therein. More recently, these systems have also become relevant as idealized models of cell activity, see [Coombes, 2008; Tonnelier, 2002; Tonnelier & Gerstner, 2003].

The case of continuous piecewise linear differential systems, when they have only two half-planes separated by a straight line is the simplest possible configuration of piecewise linear differential systems. In 1991, Lum and Chua [Lum & Chua, 1990] conjectured that a continuous piecewise linear vector field in the plane with two zones has at most one limit cycle. In 1998, this conjecture was proved by Freire, Ponce, Rodrigo and Torres in [Freire *et al.*, 1998]. We note that even in this apparent simple case, only after a hard analysis, it was possible to