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Birth of limit cycles for a class of continuous and discontinuous differential systems in $(d+2)$–dimension

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ABSTRACT
The orbits of the reversible differential system $\dot{x} = -y$, $\dot{y} = x$, $\dot{z} = 0$, with $x, y \in \mathbb{R}$ and $z \in \mathbb{R}^d$, are periodic with the exception of the equilibrium points $(0, 0, z_1, \ldots, z_d)$. We compute the maximum number of limit cycles which bifurcate from the periodic orbits of the system $\dot{x} = -y$, $\dot{y} = x$, $\dot{z} = 0$, using the averaging theory of first order, when this system is perturbed, first inside the class of all polynomial differential systems of degree n, and second inside the class of all discontinuous piecewise polynomial differential systems of degree n with two pieces, one in $y > 0$ and the other in $y < 0$. In the first case, this maximum number is $n^d(n-1)/2$, and in the second, it is n^{d+1}.

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1. Introduction and statements of the main results

Limit cycles have been used to model the behaviour of many real processes and different modern devices. In general, to prove the existence of limit cycles is a very difficult problem. One way to produce limit cycles is perturbing differential systems that have a linear centre. In this case, the limit cycles in a perturbed system bifurcate from the periodic orbits of the unperturbed centre. The search for the maximum number of limit cycles that polynomial differential systems of a given degree can have is part of *16th Hilbert's Problem*, and many contributions have been made in this direction (see, for instance, [1–3] and the references quoted therein).

Recently, the theory of limit cycles has also been studied in discontinuous piecewise differential systems. The analysis of these systems can be traced from Andronov *et al.*,[4] and still continues to receive attention by researchers. Discontinuous piecewise differential systems are a subject which have been developed very fast due to their strong applications to other branches of science. Currently, such systems are one of the connections between mathematics, physics and engineering. These systems model several phenomena in control systems, impact in mechanical systems, nonlinear oscillations and economics (see for instance, [5–10]). Recently, they have been shown to be also relevant as idealized models for biology [11] and models of cell activity.[12–14] For more details, see Teixeira [15] and all references therein.

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