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EXISTENCE OF POINCARÉ MAPS IN PIECEWISE LINEAR DIFFERENTIAL SYSTEMS IN $\mathbb{R}^N$

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In this paper we present a relationship between the algebraic notion of proper system, the geometric notion of contact point and the dynamic notion of Poincaré map for piecewise linear differential systems. This allows to present sufficient conditions (which are also necessary under additional hypotheses) for the existence of Poincaré maps in piecewise linear differential systems. Moreover, an adequate parametrization of the Poincaré maps make such maps invariant under linear transformations.

Keywords: Piecewise linear differential systems; Poincaré map.

1. Introduction and Main Results

Due to the encouraging increase in their applications (control theory [Lefschetz, 1965; Narendra & Taylor, 1973], design of electric circuits [Chua & Lin, 1990], neurobiology [FitzHugh, 1961; Nagumo *et al.*, 1962], etc ...) piecewise linear differential systems were studied early from the point of view of qualitative theory of ordinary differential equations [Andronov *et al.*, 1987]. Nowadays, a lot of papers are being devoted to these differential systems.

Also in mathematics, piecewise linear differential systems appear in a natural way between linear differential systems (whose qualitative behavior is "well known") and nonlinear differential systems (whose study is very difficult and the knowledge about them is poor, mainly in high dimension). Due to the advantage that, "the richness of dynamical behavior found in piecewise linear differential systems seems to be almost the same as general nonlinear systems", (see [Freire *et al.*, 1998; Llibre & Sotomayor, 1996; Teruel, 2000] for dimension 2 and [Carmona, 2002] for dimension 3) some dynamical conclusions can be easily obtained from their linear parts. Nevertheless, the analysis of the corresponding dynamics is far from being trivial.

In this paper, we emphasize a deep relationship that exists in piecewise linear differential systems between the algebraic notion of proper system, the geometric existence of contact points and the dynamical existence of Poincaré maps.

Consider the n-dimensional piecewise linear (differential) systems in the *Lure's form*

$$\frac{d\mathbf{x}}{ds} = \dot{\mathbf{x}} = A\mathbf{x} + \varphi(\mathbf{k}^T\mathbf{x})\mathbf{u} + \mathbf{v}\,, \qquad (1)$$

where A is an $n \times n$ real matrix, $\mathbf{k}$, $\mathbf{u}$, $\mathbf{v} \in \mathbb{R}^n$, $\mathbf{k}$ and $\mathbf{u}$ different from $\mathbf{0}$ and