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Phase portraits of the two-body problem with Manev potential

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Abstract

The Manev systems are two-body problems defined by a potential of the form $a/r + b/r^2$, where r is the distance between the two particles, and a and b are arbitrary constants. The Hamiltonian $H = (p_r^2 + p_\theta^2/r^2)/2 + a/r + b/r^2$ and the angular momentum $p_\theta = r^2\dot{\theta}$ associated with Manev systems are two first integrals, which are independent and in involution. Let I_h (respectively I_c) be the set of points of the phase space on which H (respectively p_θ) takes the value h (respectively c). Since H and p_θ are first integrals, the sets I_h, I_c and $I_{hc} = I_h \cap I_c$ are invariant under the flow of the Manev systems. We characterize the global flow of these systems when a and b vary. Thus we describe the foliation of the phase space by the invariant sets I_h and the foliation of I_h by the invariant sets I_{hc}.

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(Some figures in this article are in colour only in the electronic version; see `www.iop.org`)

1. Introduction

In this paper we describe the global phase portraits of the Manev systems (Maneff in German and French spelling); i.e. a two-body problem defined by the potential of the form $a/r + b/r^2$, where r is the distance between the two particles and a, b are arbitrary constants.

The study of the motion of these two-body problems has a long history. As early as in Newton's work, discrepancies between the observed and theoretical motions of pericentres raised the question concerning the accuracy of the Newtonian inverse square law of gravitation, and motivated the consideration of alternative gravitational models and corrections to reconcile these differences and obtain a satisfactory degree of agreement between the observational evidence and theoretical predictions on the motion of celestial bodies in the solar system; mainly the Moon, but also the planets.