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MEROMORPHIC INTEGRABILITY OF THE HAMILTONIAN SYSTEMS WITH HOMOGENEOUS POTENTIALS OF DEGREE -4

Jaume Llibre

Departament de Matemàtiques, Universitat Autònoma de Barcelona
Edifici C, 08193 Bellaterra, Barcelona, Catalonia, Spain

Yuzhou Tian*

School of Mathematics (Zhuhai), Sun Yat-Sen University
Zhuhai 519082, China

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Abstract. We characterize the meromorphic Liouville integrability of the Hamiltonian systems with Hamiltonian $H = \left(p_1^2 + p_2^2\right)/2 + 1/P(q_1, q_2)$, being $P(q_1, q_2)$ a homogeneous polynomial of degree 4 of one of the following forms $\pm q_1^4$, $4q_1^3q_2$, $\pm 6q_1^2q_2^2$, $\pm\left(q_1^2+q_2^2\right)^2$, $\pm q_2^2\left(6q_1^2-q_2^2\right)$, $\pm q_2^2\left(6q_1^2+q_2^2\right)$, $q_1^4+6\mu q_1^2q_2^2-q_2^4$, $-q_1^4+6\mu q_1^2q_2^2+q_2^4$ with $\mu > -1/3$ and $\mu \neq 1/3$, and $q_1^4+6\mu q_1^2q_2^2+q_2^4$ with $\mu \neq \pm 1/3$. We note that any homogeneous polynomial of degree 4 after a linear change of variables and a rescaling can be written as one of the previous polynomials. We remark that for the polynomial $q_1^4+6\mu q_1^2q_2^2+q_2^4$ when $\mu \in \{-5/3, -2/3\}$ we only can prove that it has no a polynomial first integral.

1. Introduction and main results.

Hamiltonian systems play an important role in the theory of the dynamical systems due to the fact that these systems occur frequently in mathematical physics, particularly in mechanics, engineering and other fields. In order to describe global information on the Hamiltonian systems is good to find sufficient number of functionally independent first integrals. The fact that a Hamiltonian system has some additional first integral independent of its Hamiltonian is a rare phenomenon which lead to a difficult problem, how to determine whether a given Hamiltonian system has additional independent first integrals.

In this work we consider the Hamiltonian systems of two degrees of freedom

$$\dot{q}_i = p_i, \quad \dot{p}_i = -\frac{\partial V}{\partial q_i}, \quad i = 1, 2, \tag{1}$$

with the Hamiltonian

$$H = \frac{1}{2}\sum_{i=1}^{2} p_i^2 + V\left(\mathbf{q}\right), \tag{2}$$

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* Corresponding author: Yuzhou Tian.