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# Symmetric periodic orbits for the collinear charged 3-body problem

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In this paper we study the existence of periodic symmetric orbits of the 3-body problem when each body possesses mass and an electric charge. The main technique applied in this study is the continuation method of Poincaré. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4974511]

## I. INTRODUCTION

One of the most relevant objects to study in the theory of dynamical systems is the $n$-body problem and many works have been done for understanding its dynamics. Thus the study of its periodic orbits is one of the main objectives. In this paper we focus the attention on the periodic orbits of the 3-body problem when the three bodies are collinear and charged.

Recently many distinct techniques and methods have been used to prove the existence of periodic orbits for the $n$-body problem, for example, averaging theory, numerical analysis, Melnikov functions, normal forms, variational methods, among others. One of the first analytical studies of the existence of periodic orbits for the $n$-body problem was done by Poincaré in Ref. 14, and we apply his method to study the symmetric periodic orbits of the charged collinear 3-body problem.

There exists a large literature studying the existence of periodic solutions of the $n$-body problem, see Refs. 11 and 12, for example. More precisely, if we restrict our attention to the 3-body problem, Hénon in Ref. 9 has studied numerically the existence and stability of a class of symmetric rectilinear periodic orbits of the general problem of three bodies. In Ref. 2 the authors studied, numerically, families of symmetric periodic orbits for the collinear 3-body problem when the two non-central masses are equal. In Ref. 10 the singularity generated by the triple collision of bodies of the collinear 3-body problem is studied.

In Ref. 5 the authors studied the symmetric periodic orbits by the continuation method of Poincaré of the collinear 3-body problem when the bodies do not have electric charges. In the present paper we allow that the bodies possess electric charges.

The continuation method of Poincaré was originally presented in Ref. 14 and this method consists in given a periodic solution for the system with a parameter equal to zero and it provides conditions for extending this solution to small values of the parameter. For more details about this method see, for example, Ref. 6.

A related problem with the collinear charged 3-body is the helium atom and the collinear models of the semiclassical theory. The collinear helium system, which consists of two electrons of mass $m_e$ and charge $-e$ moving on a line with respect to a *fixed* positively charged nucleus of charge $+2e$. The Hamiltonian modelizing the collinear helium system can be brought to the non-dimensionalized form

$$H_1 = \frac{1}{2}(\dot{r}_1^2 + \dot{r}_2^2) - \frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{r_1 + r_2},$$

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