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On the Integrability of a Tritrophic Food Chain Model

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In this paper we work with a vastly analyzed tritrophic food chain model. We provide a complete characterization of their Darboux polynomials and of their exponential factors. We also show the non–existence of polynomial first integrals, of rational first integrals, of local analytic first integrals in a neighborhood of the origin, of first integrals that can be described by formal series and of Darboux first integrals.

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Key words. Formal integrability, Darboux integrability, polynomial integrability, analytic integrability, tritrophic food chain model, Darboux polynomial, exponential factor

1 Introduction

During the last 80 years and after the works of Lotka [17] and Volterra [23] one of the main topics in mathematical ecology has been the study of ditrophic food chains

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