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Nonintegrability of a class of the Bianchi VI_0 and VII_0 models

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ABSTRACT

The well-known Bianchi VI_0 and Bianchi VII_0 dynamical systems are three-dimensional differential systems which after a convenient reduction become

$$\dot{x} = -x^2 + (z+1)y^2, \qquad \dot{y} = -4(z+1) + xyz, \qquad \dot{z} = -yz(z+2).$$

In the paper of Maciejewski and Szydiowski [A.J. Maciejewski, M. Szydiowski, Bianchi cosmologies as dynamical systems, Celestial Mech. Dynam. Astronom. 73 (1999) 17–24], the authors asked about the integrability or nonintegrability of this system. Here we show that this system has no first integrals which are polynomial, rational, Darboux functions or analytic functions. Consequently this system is not integrable inside these classes of functions.

1. Introduction and statement of the main results

Models of relativistic cosmology are based on Einstein's theory of gravitation. Einstein's field equations describe the dynamical evolution of spacetime, as well as the motion of matter and physical fields. They provide a system of coupled, nonlinear, partial differential equations, which in its full expression are not tractable by analytical tools. Therefore some simplifying assumptions are needed. The most natural one is to postulate a certain symmetry of spacetime. Usually such an idealization allows one to reduce Einstein's field equations to a system of ordinary differential equations.

The dynamics of the Bianchi VI_0 and VII_0 dynamical systems are described by three-dimensional differential equations. These two models after a convenient reduction (see [1] for details) lead to the three-dimensional system

$$\begin{aligned} \dot{x} &= -x^2 + (z+1)y^2, \\ \dot{y} &= -4(z+1) + xyz, \\ \dot{z} &= -yz(z+2), \end{aligned} \tag{1}$$

with $(x, y, z) \in \mathbb{R}^3$, and the dot denotes as usual differentiation with respect to time t.

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