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Integrability of the Bianchi IX system

Jaume Llibre[a)]
Departament de Matemàtiques, Universitat Autónoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

Clàudia Valls[b)]
Departamento de Matemática, Instituto Superior Técnico, Av. Rovisco Pais 1049-001, Lisboa, Portugal

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In this paper we apply the Darboux theory of integrability to the classical Bianchi IX system. Thus, we provide a complete description of the Darboux polynomials, exponential factors, rational first integrals, and Darboux first integrals, for the classical Bianchi IX system. © *2005 American Institute of Physics.*
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I. INTRODUCTION TO THE PROBLEM

The Bianchi IX model can be written as the Hamiltonian system (see Ref. 8):

$$\dot{q}_1 = 12q_1(p_1q_1 - p_2q_2 - p_3q_3),$$

$$\dot{q}_2 = 12q_2(-p_1q_1 + p_2q_2 - p_3q_3),$$

$$\dot{q}_3 = 12q_3(-p_1q_1 - p_2q_2 + p_3q_3),$$

$$\dot{p}_1 = -12p_1(p_1q_1 - p_2q_2 - p_3q_3) - \tfrac{1}{3}(q_1 - q_2 - q_3),$$

$$\dot{p}_2 = -12p_2(-p_1q_1 + p_2q_2 - p_3q_3) - \tfrac{1}{3}(-q_1 + q_2 - q_3),$$

$$\dot{p}_3 = -12p_3(-p_1q_1 - p_2q_2 + p_3q_3) - \tfrac{1}{3}(-q_1 - q_2 + q_3), \tag{1}$$

in $\mathbb{R}^6$ with the Hamiltonian

$$G = 6(p_1^2q_1^2 + p_2^2q_2^2 + p_3^2q_3^2 - 2p_1q_1p_2q_2 - 2p_1q_1p_3q_3 - 2p_2q_2p_3q_3) + \frac{1}{6}(q_1^2 + q_2^2 + q_3^2 - 2q_1q_2 - 2q_1q_3 - 2q_2q_3).$$

This system has been throughly investigated from the point of view of integrability by using different methods for studying the existence of first integrals. One of these methods is the Painlevé test (see, for instance, Refs. 10, 5, and 6) where the authors prove that the solutions of the equation of motion do not have movable critical points and conjecture that system (1) is integrable. Later on, in Ref. 7 the same authors of Ref. 6 find new arguments, this time against the integrability. Another method is the numerical computation of the Lyapunov exponents (see Ref. 1), where the authors also conjecture the integrability of system (1). In Ref. 8 the authors, by means of sym-

[a)]Electronic mail: jllibre@mat.uab.es
[b)]Electronic mail: cvalls@math.ist.utl.pt

