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Formal and analytic first integrals of the Einstein–Yang–Mills equations

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Abstract
In this paper we provide a complete description of the first integrals of the classical Einstein–Yang–Mills equations that can be described by formal series. As a corollary we also obtain a complete description of the analytic first integrals in a neighbourhood of the origin.

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1. Introduction to the problem

The static, spherically symmetric Einstein–Yang–Mills equations [1–3, 12, 13] with a cosmological constant $a \in \mathbb{R}$ are given by the differential system

$$\begin{aligned} \dot r &= rN, & \dot W &= rU, \\ \dot N &= (k-N)N - 2U^2, & \dot k &= s(1-2ar^2) + 2U^2 - k^2, \\ \dot U &= sWT + (N-k)U, & \dot T &= 2UW - NT, \end{aligned} \tag{1}$$

where $r, W, N, k, U, T \in \mathbb{R}^6$ and $s \in \{-1; 1\}$, and the dot denotes a derivative with respect to the space–time variable t.

Let

$$f = 2kN - N^2 - 2U^2 - s(1 - T^2 - ar^2). \tag{2}$$

Then, over the solutions $(r(t), W(t), N(t), k(t), U(t), T(t))$ of system (1) it holds

$$\frac{\mathrm{d}f(t)}{\mathrm{d}t} = -2N(t)f(t).$$

Therefore, we obtain that $f = 0$ is an invariant hypersurface under the flow of system (1); i.e., if a solution of system (1) has a point on $f = 0$ the whole solution is contained in $f = 0$.

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