

Formal and analytical integrability of the Bianchi IX system

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In this paper we provide a complete description of the first integrals of the classical Bianchi IX system that can be described by a general class of formal power series. As a corollary we also obtain a complete description of some of its analytic first integrals in a neighborhood of the origin. In particular, we prove that the system is not completely integrable by analytic first integrals. © 2006 American Institute of Physics. [DOI: [10.1063/1.2168123](https://doi.org/10.1063/1.2168123)]

I. INTRODUCTION TO THE PROBLEM

The Bianchi IX model can be written as the Hamiltonian system (see Ref. 4):

$$\begin{aligned}\dot{q}_1 &= F_1(q, p) = 12q_1(p_1q_1 - p_2q_2 - p_3q_3), \\ \dot{q}_2 &= F_2(q, p) = 12q_2(-p_1q_1 + p_2q_2 - p_3q_3), \\ \dot{q}_3 &= F_3(q, p) = 12q_3(-p_1q_1 - p_2q_2 + p_3q_3), \\ \dot{p}_1 &= F_4(q, p) = -12p_1(p_1q_1 - p_2q_2 - p_3q_3) - \frac{1}{3}(q_1 - q_2 - q_3), \\ \dot{p}_2 &= F_5(q, p) = -12p_2(-p_1q_1 + p_2q_2 - p_3q_3) - \frac{1}{3}(-q_1 + q_2 - q_3), \\ \dot{p}_3 &= F_6(q, p) = -12p_3(-p_1q_1 - p_2q_2 + p_3q_3) - \frac{1}{3}(-q_1 - q_2 + q_3),\end{aligned}\tag{1}$$

with the Hamiltonian

$$\begin{aligned}H &= 6(p_1^2q_1^2 + p_2^2q_2^2 + p_3^2q_3^2 - 2p_1q_1p_2q_2 - 2p_1q_1p_3q_3 - 2p_2q_2p_3q_3) \\ &\quad + \frac{1}{6}(q_1^2 + q_2^2 + q_3^2 - 2q_1q_2 - 2q_1q_3 - 2q_2q_3).\end{aligned}$$

Of course, here (q, p) denotes $(q_1, q_2, q_3, p_1, p_2, p_3)$.

The solutions which lie in the zero level set of the Hamiltonian H , provide a model for describing the evolution of the universe, see for instance, Ref. 6. Furthermore, the set defined as

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