FORMAL AND ANALYTIC INTEGRABILITY OF THE BELOUSOV–ZHABOTINSKII SYSTEM

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Abstract. The well–known Belousov–Zhabotinskii system can be written as $\dot{x} = s(x+y-qx^2-xy)$, $\dot{y} = s^{-1}(-y+fz-xy)$ and $\dot{z} = w(x-z)$. Here we study the first integrals of the Belousov–Zhabotinskii system that can be described by formal power series.

1. Introduction and statement of the main results

The Belousov–Zhabotinskii system (see [1]):

$$\dot{x} = s(x+y-qx^2-xy), \quad \dot{y} = s^{-1}(-y+fz-xy), \quad \dot{z} = w(x-z), \tag{1}$$

is one of the most interesting and best understood dynamical oscillators, where x, y, z are real variables, and s, f, q, w are real parameters. This system has been intensively investigated as a dynamical system (see for instance [3, 4, 8, 12, 13, 14]).

The associated vector field of the Belousov–Zhabotinskii system is

$$X = s(x+y-qx^2-xy)\frac{\partial}{\partial x} + s^{-1}(-y+fz-xy)\frac{\partial}{\partial y} + w(x-z)\frac{\partial}{\partial z}.$$

Let U be an open subset of $\mathbb{R}^3$ and let $g : U \to \mathbb{R}$ be a C^1 function. Then g is a *first integral* of system (1) if it is not a constant function and satisfies

$$Xg = s(x+y-qx^2-xy)\frac{\partial g}{\partial x} + s^{-1}(-y+fz-xy)\frac{\partial g}{\partial y} + w(x-z)\frac{\partial g}{\partial z} = 0.$$

Therefore, the function g is constant over the solutions of the Belousov–Zhabotinskii system (1) contained in U.

In this paper a *formal first integral* $g = g(x,y,z)$ of system (1) is a non–constant formal power series in the variables x, y and z satisfying $Xg = 0$. Note that if the power series is convergent in an open subset U of $\mathbb{R}^3$, then the analytic function defined by it is a first integral in U of system (1).

1991 *Mathematics Subject Classification.* 34C05, 34A34, 34C14.

Key words and phrases. formal integrability, analytic integrability, Belousov–Zhabotinskii system.