

On the integrability of the Einstein–Yang–Mills equations

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Abstract

In this paper we apply the Darboux theory of integrability to the Einstein–Yang–Mills differential equations. Thus, we provide a complete description of the Darboux polynomials, exponential factors, rational first integrals and Darboux first integrals for the Einstein–Yang–Mills differential equations.

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1. Introduction to the problem

The static, spherically symmetric Einstein–Yang–Mills equations [1–3,12,13] with a cosmological constant $a \in \mathbb{R}$ are given by the differential system

$$\dot{r} = rN,$$

$$\dot{W} = rU,$$

$$\dot{N} = (k - N)N - 2U^2,$$

$$\dot{k} = s(1 - 2ar^2) + 2U^2 - k^2,$$

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