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FORMAL AND ANALYTIC INTEGRABILITY OF THE ROSSLER SYSTEM

JAUME LLIBRE
Departament de Matemàtiques,
Universitat Autònoma de Barcelona,
08913 Bellaterra, Barcelona, Spain
jllibre@mat.uab.es

CLÀUDIA VALLS
Departamento de Matemática,
Instituto Superior Técnico, 1049–001 Lisboa, Portugal
cvalls@math.ist.utl.pt

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The well–known Rossler system can be written as $\dot{x} = -(z+y)$, $\dot{y} = x + ay$ and $\dot{z} = b + xz - cz$. Here, we study the first integrals of the Rossler system that can be described by formal power series.

Keywords: Integrability; polynomial systems.

1. Introduction and Statement of the Main Results

It is not easy to try to understand what are the integrable systems. The question can occupy an entire book [Zakharov, 1991]. Roughly speaking we can say that integrability of a system of differential equations should manifest itself through some generally recognizable features as (1) the existence of conserved quantities, (2) the presence of algebraic geometry, and (3) the ability to give explicit solutions, see [Hitchin, 1997] for more details. Here, we will work with the notion of integrability associated with the existence of conserved quantities, more precisely to the existence of first integrals. Of course, when a system presents first integrals, they help strongly to understand the dynamics of the system. So it is important to know when a system has or not first integrals, and what are their nature, analytical, smooth, ...

The Rossler system:

$$\dot{x} = -(z+y), \quad \dot{y} = x + ay, \quad \dot{z} = b + xz - cz, \quad (1)$$

is a famous dynamical model, where x, y and z are real variables; and a, b and c are real parameters, see [Rossler, 1976]. Related with this system there are more than 300 papers published (see MathSciNet) in which mainly the notion of dynamical chaos is investigated .

A *formal first integral* of the Rossler system (1) is a nonconstant formal power series which satisfies that $Xf = 0$, where X is the vector field associated with the Rossler system, that is,

$$X = -(z+y)\frac{\partial}{\partial x} + (x+ay)\frac{\partial}{\partial y} + (b + xz - cz)\frac{\partial}{\partial z}.$$

The main results of this paper are the following. The first one deals with system (1) restricted to $b = 0$.

Theorem 1.1. *The following holds for system* (*1*) *with* $b = 0$.

(a) *If* $a = c = 0$, *then system* (*1*) *is integrable with the first integrals*

$$H_1 = x^2 + y^2 + 2z, \quad H_2 = ze^{-y}.$$