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Bull. Sci. math. 132 (2008) 40–53

BULLETIN DES SCIENCES MATHÉMATIQUES

www.elsevier.com/locate/bulsci

Centers for a 6-parameter family of polynomial vector fields of arbitrary degree

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Received 24 May 2007

Available online 7 June 2007

Abstract

For all non-negative integers n_1, n_2, n_3, j_1, j_2 and j_3 with $n_k + j_k > 1$ for $k = 1, 2, 3$, $(n_k, j_k) \neq (n_l, j_l)$ if $k \neq l$, $j_3 = n_3 - 1$ and $j_k \neq n_k - 1$ for $k = 1, 2$, we study the center variety of the 6-parameter family of real planar polynomial vector $(\dot{x}, \dot{y})$ given, in complex notation, by $\dot{z} = iz + Az^{n_1}\bar{z}^{j_1} + Bz^{n_2}\bar{z}^{j_2} + Cz^{n_3}\bar{z}^{j_3}$, where $z = x + iy$ and $A, B, C \in \mathbb{C}\backslash\{0\}$.

MSC: primary 34C05; secondary 58F14

Keywords: Centers; Polynomial vector fields; Reversibility

1. Introduction and statement of the main results

One of the main problems in the qualitative theory of real planar differential systems is the determination of *centers*. A *center* is a singular point $p \in \mathbb{R}^2$ having a neighborhood U such that all the orbits of $U \setminus \{p\}$ are periodic. The notion of center was introduced by Poincaré in [14].

In what follows and without loss of generality we assume that the singular point candidate to be a center is located at the origin of $\mathbb{R}^2$. A *non-degenerate* center (i.e. a center having eigenvalues of the form $\pm\beta i$ with $\beta \neq 0$). An usual method for looking for a non-degenerate center of a family of planar polynomial vector fields is to calculate the successive coefficients v_i of the return map of the vector field about the origin. That is, we choose a segment $(0, x_0]$ on the

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0007-4497/$ – see front matter
doi:10.1016/j.bulsci.2007.06.002