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Integrability of a SIS model

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Abstract

We prove that the classical model of an infectious disseise, which never kills and which does not induce autoimmunity, is integrable. This model can be written as $x' = -bxy - mx + cy + mk$, $y' = bxy - (m + c)y$ with parameters $b, c, k, m \in \mathbb{R}$. We provide the explicit expression of its first integrals and of the set of all its invariant algebraic curves.

Keywords: Darboux integrability; SIS equation; Darboux polynomials; Invariant algebraic curves; Exponential factors

1. Introduction

In this paper we consider a simple epidemiological model which is a particular case of the classical SIS model introduced by Kernack and McKendrick in [7] and which is given by the following differential equation (see [2] for details):

$$x' = -bxy - mx + cy + mk, \qquad y' = bxy - (m + c)y, \tag{1}$$

where $b, c, k, m \in \mathbb{R}$ and the prime indicates derivative with respect to the time. We note that $x = x(t)$ is the susceptible component of the population, $y = y(t)$ is the infected component of the population, mk is the constant birth rate, m is the proportionate death rate, b is the infectivity coefficient of the typical Lotka–Volterra interaction term, and c is the recovery coefficient. We note that the disease is assumed to be nonfatal so that the standard term removing deceased infectaves $-ay$ in reference [2] is omitted.

Of course the polynomial differential system (1) is defined in the whole plane $\mathbb{R}^2$. We only consider the differential system (1) with $b \neq 0$ and $m \neq 0$, because if $b = 0$ the system is linear and it can be solved explicitly, and if $m = 0$, then the system has the invariant straight line $y = 0$ of singular points and doing a rescaling of the time variable it becomes again linear. Moreover these two cases, $b = 0$ or $m = 0$, have no interest as SIS models.

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