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Global analytic integrability of the Rabinovich system

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ABSTRACT

The Rabinovich system can be written as $\dot{x} = hy - v_1x + yz$, $\dot{y} = hx - v_2y - xz$ and $\dot{z} = -v_3z + xy$ with h, v_1, v_2 and v_3 being real parameters. In this paper we characterize its global analytic first integrals.

1. Introduction and statement of the main results

The Rabinovich differential system is defined by the following four parameter family of quadratic differential equations

$$\begin{aligned} \dot{x} &= hy - v_1x + yz, \\ \dot{y} &= hx - v_2y - xz, \\ \dot{z} &= -v_3z + xy, \end{aligned} \tag{1}$$

in $\mathbb{R}^3$ with variables (x, y, z) and with the parameters $(h, v_1, v_2, v_3) \in \mathbb{R}^4$. As usual the dots denote derivatives with respect to the time t.

System (1) was firstly studied in [8] with the analysis of a concrete realization in a magnetoactive non-isothermal plasma. From the physical point of view, system (1) is a dynamical system of three resonantly coupled waves, parametrically excited (see for more details [7] or [1]). Through numerical investigations they obtain values of the parameters for which a strange attractor, very similar to the Lorenz attractor, is produced and corresponds to stochastic self-oscillations of the wave amplitudes. So the geometric studies made for the Lorenz system can be extended to the Rabinovich system.

The integrability of the Rabinovich system has been studied by different authors with different tools. Thus using the Painlevé method in 1984, Bountis et al. [1] found three integrals of motion for particular values of the parameters. By using some algebraic methods in 1991, Giacomini et al. [2] obtained more additional four integrals again for particular values of the parameters. In 2000 Zhang [10] and in 2003 Xie and Zhang [9] studied the Darboux integrability of the Rabinovich systems. Here we shall study its analytical integrability.

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