Analytic integrability of a Chua system

Jaume Llibre[1,a)] and Clàudia Valls[2,b)]
[1]*Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, Barcelona, Catalonia 08193, Spain*
[2]*Departamento de Matemática, Instituto Superior Técnico, 1049-001 Lisboa, Portugal*

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We consider the system $\dot{x}=a(z-a_1x^3-a_2x^2-bx)$, $\dot{y}=-z$, $\dot{z}=-b_1x+y+b_2z$, where a and b are parameters and $b_1=7/10$, $b_2=6/25$, $a_1=44/3$, and $a_2=41/2$. We analyze the existence of local and global analytic first integrals. © *2008 American Institute of Physics.* [DOI: 10.1063/1.2992481]

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

One of the more classical problems in the qualitative theory of real analytic differential systems in $\mathbb{R}^3$ is to characterize the existence of first integrals in a neighborhood of an isolated singular point and, in general, the existence of local and global analytic first integral.

Let U be an open subset of $\mathbb{R}^3$, $H:U\to\mathbb{R}$ be a nonconstant analytic function, and $\mathcal{X}$ be an analytic vector field defined on $\mathbb{R}^3$. Then H is an *analytic first integral* of $\mathcal{X}$ in U if H is constant on the solutions of $\mathcal{X}$ contained in U; i.e., if $\mathcal{X}H=0$ in U. If U is a neighborhood of a singular point p, then H is called a *local analytic first integral* of $\mathcal{X}$ at p. If $U=\mathbb{R}^3$, then H is called a *global analytic first integral* of $\mathcal{X}$.

We consider the system

$$\dot{x}=a(z-a_1x^3-a_2x^2-bx), \quad \dot{y}=-z, \quad \dot{z}=-b_1x+y+b_2z, \tag{1}$$

where a and b are parameters and $b_1=7/10$, $b_2=6/25$, $a_1=44/3$, and $a_2=41/2$. For some values of the parameters a and b we characterize the existence of local analytic first integrals of system (1) at the singular point located at the origin of coordinates, and also we determine the global analytic first integrals of system (1).

The Chua circuit[1,5] is a relaxation oscillator with a cubic nonlinear characteristic elaborated from a circuit comprising a harmonic oscillator for which the operation is based on a field-effect transistor, coupled to a relaxation oscillator composed of a tunnel diode. The modeling of the circuit uses a capacity which will prevent abrupt voltage drops and makes it possible to describe the fast motion of this oscillator by the system (1) for convenient values of the parameters a and b.

The main result in this paper is the following theorem. To state it, we introduce some notation. Set

$$a=\frac{a_\pm}{25b(12b-35)} \quad \text{and} \quad b>0, \tag{2}$$

where

$$a_\pm=-105+36b\pm\sqrt{21}\sqrt{525+12140b-4224b^2},$$

and set

[a)]Electronic mail: jllibre@mat.uab.cat.
[b)]Electronic mail: cvalls@math.ist.utl.pt.

