J. Differential Equations 246 (2009) 2192–2204

Contents lists available at ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

Classification of the centers, their cyclicity and isochronicity for a class of polynomial differential systems generalizing the linear systems with cubic homogeneous nonlinearities

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ARTICLE INFO

Article history:
Received 7 May 2007
Revised 6 November 2008
Available online 15 January 2009

ABSTRACT

In this paper we classify the centers, the cyclicity of its Hopf bifurcation and their isochronicity for the polynomial differential systems in $\mathbb{R}^2$ of arbitrary degree $d \geqslant 3$ odd that in complex notation $z = x + iy$ can be written as

$$\dot{z} = (\lambda + i)z + (z\bar{z})^{\frac{d-3}{2}}\left(Az^3 + Bz^2\bar{z} + Cz\bar{z}^2 + D\bar{z}^3\right),$$

where $\lambda \in \mathbb{R}$ and $A, B, C, D \in \mathbb{C}$. If $d = 3$ we obtain the well-known class of all polynomial differential systems of the form a linear system with cubic homogeneous nonlinearities.

1. Introduction and statement of the main results

Probably two of the main problems in the qualitative theory of real planar polynomial differential systems are the determination of limit cycles and the center–focus problem; i.e. to distinguish when a singular point is either a focus or a center. The notion of *center* goes back to Poincaré in [19]. He defined it for a vector field on the real plane; i.e. a singular point surrounded by a neighborhood fulfilled of closed orbits with the unique exception of the singular point. This paper deals with the center–focus problem for a class of polynomial differential systems which generalizes the class of cubic polynomial differential systems with homogeneous nonlinearities.

The classification of the centers of the polynomial differential systems started with the quadratic ones with the works of Dulac [7], Kapteyn [12,13], Bautin [2], etc. Schlomiuk, Guckenheimer and Rand

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0022-0396/$ – see front matter
doi:10.1016/j.jde.2008.12.006