

GLOBAL ANALYTIC FIRST INTEGRALS FOR THE SIMPLIFIED MULTISTRAIN/TWO-STREAM MODEL FOR TUBERCULOSIS AND DENGUE FEVER

JAUME LLIBRE^{*,‡} and CLÀUDIA VALLS^{†,§}

**Department de Matemàtiques
Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Spain*

*†Departamento de Matemática
Instituto Superior Técnico
Av. Rovisco Pais 1049-001, Lisboa, Portugal
[‡]jllibre@mat.uab.cat
[§]cvals@math.ist.utl.pt*

Received 14 April 2009

Accepted 10 June 2009

We provide the complete classification of all global analytic first integrals of the simplified multistrain/two-stream model for tuberculosis and dengue fever that can be written as

$$\dot{x} = x(\beta_1 - b - \gamma_1 - \beta_1 x - (\beta_1 - \nu)y), \quad \dot{y} = y(\beta_2 - b - \gamma_2 - (\beta_2 + \nu)x - \beta_2 y),$$

with $\beta_1, \beta_2, b, \gamma_1, \gamma_2, \nu \in \mathbb{R}$.

Keywords: Analytic first integral; multistrain/two-stream model; tuberculosis; dengue fever.

1. Introduction

The nonlinear ordinary differential equations or simply the differential systems appear in many branches of applied mathematics, physics, and in general in applied sciences. Since generically the differential systems cannot be solved explicitly, the qualitative information provided by the theory of dynamical systems is, in general, the best that one can expect to obtain.

For a two-dimensional differential system the existence of a first integral determines completely its phase portrait, i.e. the description of the domain of definition of the differential system as union of all the orbits or trajectories of the system. To provide the phase portrait of a differential system is the main objective of the qualitative theory of the differential systems. Thus for two-dimensional differential systems one of the main questions is: How to recognize if a given planar differential system has a first integral?

In this paper we characterize for a planar differential system depending on six parameters what are the values of these parameters for which there exists a global analytic first integral. For those systems having such a first integral it is possible to describe their phase portraits,