JOURNAL OF MATHEMATICAL PHYSICS **51**, 092702 (2010)

The Bianchi VIII model is neither global analytic nor Darboux integrable

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(Received 19 April 2010; accepted 9 July 2010; published online 1 September 2010)

We consider the Bianchi VIII model. This model has been studied during these past years but very few are known up to now on its integrability. We show that the Bianchi VIII system has neither a global analytic first integral nor a Darboux first integral, which is not a function of its Hamiltonian. © *2010 American Institute of Physics.* [doi:10.1063/1.3475537]

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

When cosmological models are modelized by differential systems through a set of ordinary differential equations, they can be studied using the rich theory of finite dynamical systems. However, the differential systems coming from cosmological models present many special features that distinguish them from the typical differential system.

The Bianchi relativistic homogeneous cosmological models are presented in four dimensional manifolds, three for the space and one for the time. Here, we continue the study of the integrability or nonintegrability of the Bianchi VIII models, which can be formulated as a Hamiltonian system with three degrees of freedom with Hamiltonian,

$$H = H(P_1,P_2,P_3,Q_1,Q_2,Q_3) = -P_1^2Q_2^2 - Q_1^2(1+P_2^2) - 2Q_1P_1(Q_2P_2 - Q_3P_3) + Q_2Q_3(2P_2P_3 - 1) - \frac{1}{4}Q_3^2(1+4P_3^2)$$

(see Ref. 5 for more details). In this paper, the authors proved that the Bianchi VIII model as a Hamiltonian system is not completely integrable with meromorphic first integrals (for a precise statement of their results, see Theorem 2 in Ref. 5).

Doing convenient noncanonical changes of variables (see Ref. 5 for more details), the Hamiltonian system of the Bianchi VIII model can be written as

$$\dot{y}_1 = \frac{1}{2}(y_1z_2 + y_2z_1),$$

$$\dot{y}_2 = \frac{1}{2}(y_1z_1 + y_2z_2),$$

$$\dot{y}_3 = y_3z_3,$$

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