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The Michelson system is neither global analytic, nor Darboux integrable

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ABSTRACT

We consider the differential system $\dot{x} = y, \dot{y} = z, \dot{z} = c^2 - y - x^2/2$ in $\mathbb{R}^3$, where c is a real parameter. This differential system is known as the Michelson system and its dynamics has been studied during these last twenty five years but nothing was known up to now on its integrability. We show that for any value of c the Michelson system is neither global analytic, nor Darboux integrable.

1. Introduction and statement of the main results

One of the more classical problems in the qualitative theory of real polynomial differential systems in $\mathbb{R}^3$ is to characterize the existence of first integrals. The knowledge of two independent first integrals allows to describe the phase space of the system; i.e. its qualitative behavior. The knowledge of a unique first integral can reduce in one dimension the study of the dynamics of the system.

Assume that U is an open and dense subset of $\mathbb{R}^3$, $H : U \to \mathbb{R}$ is a nonconstant function at less of class C^1, and $\mathcal{X}$ is a polynomial vector field in $\mathbb{R}^3$. Then H is a *first integral* of $\mathcal{X}$ in U if H is constant on the solutions of $\mathcal{X}$ contained in U; i.e. if $\mathcal{X}H = 0$ in U.

We consider the polynomial differential system

$$\dot{x} = y, \qquad \dot{y} = z, \qquad \dot{z} = c^2 - y - \frac{x^2}{2}, \tag{1}$$

where c is a real parameter. As usual the dot denotes a derivative with the independent variable t. This system is called the *Michelson system* and was obtained by Michelson [1] in the study of the traveling wave solutions of the Kuramoto–Sivashinsky equation. The dynamics of this system has been intensively studied; see for instance [1–5], for details and references therein. However up to now nothing has been known about its integrability. The aim of this paper is to cover this gap and to study the analytic and the Darboux integrability of the Michelson system. Of course this last kind of integrability will be studied by using the Darboux theory of integrability; see for instance [6,7].

As before let U be an open and dense subset of $\mathbb{R}^3$. An *analytic* first integral of $\mathcal{X}$ in U is a first integral H which is an analytic function in U. Moreover, if $U = \mathbb{R}^3$ then H is called a *global analytic* first integral of $\mathcal{X}$. For instance if H is an analytic first integral and a polynomial, then H is a global analytic first integral. Such first integrals are called *polynomial* first integrals.

If now we choose U as a neighborhood of a singular point p of $\mathcal{X}$ and $H : U \to \mathbb{R}$ is an analytic first integral in U, then H is called a *local analytic* first integral of $\mathcal{X}$ at p.

The main result of this paper concerning analytic first integrals is the following one.

Theorem 1. *For all $c \in \mathbb{R}$ the Michelson system (1) has no local analytic first integrals at the singular point $(-\sqrt{2}c, 0, 0)$, and consequently it has no global analytic first integrals.*

Theorem 1 is proved in Section 2.

As a corollary of Theorem 1 we have:

Corollary 2. *For all $c \in \mathbb{R}$ the Michelson system (1) has no polynomial first integrals.*

Another class of functions different from the global analytic ones but having an intersection with them is the class of the Darboux functions. The study of the Darboux first integrals for polynomial differential systems is a classical problem of the

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