Centers and Isochronous Centers for Two Classes of Generalized Seventh and Ninth Systems

Jaume Llibre · Clàudia Valls

Received: 6 July 2009 / Published online: 25 June 2010 © Springer Science+Business Media, LLC 2010

Abstract We classify new classes of centers and of isochronous centers for polynomial differential systems in \mathbb{R}^2 of arbitrary odd degree $d \ge 7$ that in complex notation z = x + iy can be written as

$$\dot{z} = (\lambda + i)z + (z\overline{z})^{\frac{d-7-2j}{2}} \left(A z^{5+j} \overline{z}^{2+j} + B z^{4+j} \overline{z}^{3+j} + C z^{3+j} \overline{z}^{4+j} + D \overline{z}^{7+2j} \right),$$

where *j* is either 0 or 1, $\lambda \in \mathbb{R}$ and *A*, *B*, *C* $\in \mathbb{C}$. Note that if *j* = 0 and *d* = 7 we obtain a special case of seventh polynomial differential systems which can have a center at the origin, and if *j* = 1 and *d* = 9 we obtain a special case of ninth polynomial differential systems which can have a center at the origin.

Keywords Centers · Isochronous · Polynomial vector fields

1 Introduction and Statement of the Main Results

In this paper we consider the polynomial differential systems in the real (x, y)-plane that has a singular point at the origin with eigenvalues $\lambda \pm i$ and that can be written as

$$\dot{z} = (\lambda + i)z + (z\overline{z})^{\frac{d-7-2j}{2}} \left(Az^{5+j}\overline{z}^{2+j} + Bz^{4+j}\overline{z}^{3+j} + Cz^{3+j}\overline{z}^{4+j} + D\overline{z}^{7+2j} \right), \quad (1)$$

where *j* is either 0 or 1, z = x + iy, $d \ge 7$ is an arbitrary odd positive integer, $\lambda \in \mathbb{R}$ and *A*, *B*, $C \in \mathbb{C}$. When j = 0 we are considering the class of systems

$$\dot{z} = (\lambda + i)z + (z\overline{z})^{\frac{d-7}{2}} \left(A z^5 \overline{z}^2 + B z^4 \overline{z}^3 + C z^3 \overline{z}^4 + D \overline{z}^7 \right),$$

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain e-mail: jllibre@mat.uab.cat

C. Valls

J. Llibre (🖂)

Departamento de Matemática, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal e-mail: cvalls@math.ist.utl.pt